Dimension Reduction of Network Bottleneck Bandwidth Data Space

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***Abstract*—The network proximity metrics, such as bottleneck bandwidth and round-trip time, are very useful in different network applications. The round-trip-time prediction has been studied extensively. However, the prediction of bottleneck bandwidth has received much less attention. Therefore, we attempt to design a new bottleneck bandwidth prediction system by matrix factorization. As a first step, we focus on the dimension reduction of network bottleneck bandwidth data space in this paper. Evaluation is carried out based on real-world bottleneck bandwidth datasets, which are collected in the past three months. The results show that a 250D data space can be compressed to 10D and the average median-relative-error is only 8.65%. Although preliminary, our work provides some insights into the design direction towards matrix factorization based distributed system to predict the bottleneck bandwidth.**

I. Introduction

Network proximity metric is helpful for improving the performance of different network services and applications, such as dynamic server selection and peer-to-peer networks. Bottleneck bandwidth and round-trip-time are important proximity metrics. Previous works, such as [1], focused on providing a scalable way to predict round-trip-time between Internet hosts. However, the prediction of bottleneck bandwidth has received much less attention. Key *et al.* [2] studied how to find the widest path based on the quantitative relationship of the bottleneck bandwidth between pairs of hosts, but they did not consider the prediction accuracy of the bottleneck bandwidth itself.

We attempt to apply the method of matrix factorization [3] in designing a new system to predict bottleneck bandwidth. In this paper, after modeling the bottleneck bandwidth data space as a large matrix whose elements represent pairwise bottleneck bandwidth, we explore the feasibility of matrix factorization by analyzing the low rank nature of the matrix and the performance of dimension reduction. Principle Component Analysis (PCA) is applied on the bottleneck bandwidth matrix. It is proved that the matrix can be projected into a much-smaller-dimension space by keeping only those few large singular values, while introducing little error. Our evaluations show that the dimension of bandwidth matrix can be reduced to 10D, and the average of median relative error in compression is only 8.65% among the 491 real-world datasets that we collected on the Internet from September to December 2009. These results provide an encouraging motivation for our future work on finishing the bottleneck bandwidth prediction system based on matrix factorization.

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II. Dimension Reduction

Suppose that N hosts exist: $H_1, H_2, ..., H_N$. The network bottleneck bandwidth data space can be modeled as a $N \times N$ matrix B. Let b_{ij} denote the bottleneck bandwidth from H_i to H_j. If the link from H_i to H_j is unavailable, b_{ij} is defined as -1.

In order to explore the low rank nature of matrix B, we apply PCA on B to factorize it. The result is:

$$B = USV^T$$

where S is diagonal with ordered singular values.

Due to PCA's advantageous property, the i^{th} singular value entirely determines the magnitude of the contribution of dimension i to the overall variance of matrix B. Therefore, if the first k singular values are much larger than the rest, we can keep only the first k singular values, resulting in a $k \times k$ matrix S' ($k \ll N$). Keeping only the first k columns of matrix U to form U' (the same operation on V to get V'), the bandwidth matrix B is approximated by B':

$$B \approx B' = U'S'V'^T = (U'\sqrt{S'})(V'\sqrt{S'})^T = XY^T$$

where X and Y are $N \times k$ matrices.

It is also proven that PCA can minimize the following error measurement function $f_{error}(B, B')$:

$$f_{error}(B, B') = \sum_{(i,j) \in \{(m,n) | b_{mn} \neq -1\}} (b_{ij} - b'_{ij})^2$$

This way, the dimension of the network bottleneck bandwidth data space is reduced from N to k, while only introducing marginal error from the perspective of $f_{error}(B, B')$. A host H_i can be represented by two k-dimension row vectors, X_i as *outgoing vector* and Y_i as *incoming vector*. The bottleneck bandwidth from H_i to H_j (b_{ij}) can be computed by $X_i Y_j^T$.

In the next section, the performance of the dimension reduction method is evaluated.

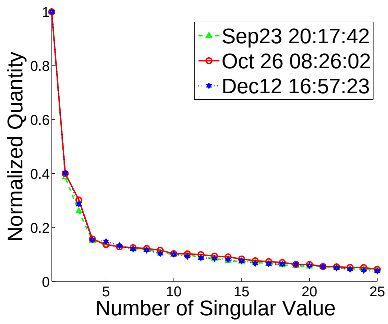

Fig. 1. Singular Values

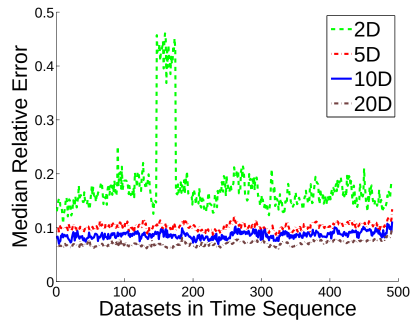

Fig. 2. Median Relative Error

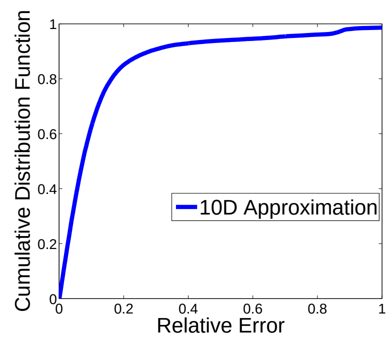

Fig. 3. Cumulative Distribution Function

III. Numeric Evaluation

A. Intrinsic Dimensionality Analysis

Experiments are carried out based on HP *Scalable Sensing Service (S3)* data. *S3* is deployed on 453 PlanetLab hosts to measure the bandwidth between pairs of hosts. There are some pairs of hosts in *S3*, where the links in both directions are unavailable. A greedy algorithm is used to eliminate these pairs. As a result, 250 hosts are extracted out of all the 453 ones for our evaluation.

From September 23^{rd} to December 23^{rd} 2009, we collected the bandwidth data every four hours, finally getting 491 datasets. Our PCA analysis on all these 491 datasets shows that the first few singular values are much larger than the rest. Since the evaluation results are similar, we randomly pick three datasets, which are collected at 20:17:42 Sep 23^{rd} 2009, 08:26:02 Oct 26^{th} 2009 and 16:57:23 Dec 12^{th} 2009. In Figure 1, the singular values are plotted in decreasing order (the largest one is normalized to 1). It shows that singular values decrease very fast. Considering the 'Oct 26' line, the 4^{th} singular value (0.156) is the first one that is smaller than 0.2. The index is much smaller than the number of the matrices' columns (250). It verifies our idea that the bottleneck bandwidth matrix has a low rank nature.

B. Relative Error in Dimension Reduction

In our evaluation, relative error is defined as follows. If $(i,j) \in \{(m,n)|b_{mn} \neq -1\}$, $relative\ error_{ij} = |b'_{ij} - b_{ij}|/b_{ij}$. *Median relative error* is defined as $median_{i,j}(relative\ error_{ij})$. For every dataset, the dimension of data space (250D) is reduced to a variable, ranging from 1D to 30D. In each process of dimension reduction, we analyze the relative error between the entries in approximated matrix B' and original matrix B. The result shows that the median relative error decreases to lower than 10% when the target dimension of reduction is larger than 5D. Figure 2 shows the median relative error of all the datasets when their dimensions are reduced to 2D, 5D, 10D and 20D. The average of median relative error for 10D approximation is only 8.65% among all the 491 datasets.

The 90^{th} percentile relative error (NPRE) is also used to indicate the distribution of relative error in approximation. It guarantees 90% of the data have lower relative error than the value of NPRE. Considering the tradeoff between computation complexity and target dimension of reduction, a 10D approximation is carried out on a dataset to show the cumulative distribution function of relative error, which is collected at 04:27:22 Oct 30^{th} 2009. Figure 3 shows that NPRE is only 0.281.

IV. Conclusion and Future Work

We studied the problem of dimension reduction of network bottleneck bandwidth data space. By applying PCA on extensive data from PlanetLab, we proved the low rank nature of bottleneck bandwidth matrix. By keeping only those few large singular values, the dimension of network bottleneck bandwidth data space can be greatly reduced while introducing little relative error and low NPRE.

The low rank nature and the low relative error in dimension reduction verifies the feasibility of our idea on designing a bottleneck bandwidth prediction system based on matrix factorization. Our future work is to implement the prediction system and deploy it on the Internet for live network evaluation.

References

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